The life cycle model of individual choices: a unifying framework

Orazio P. Attanasio

University College London, Institute for Fiscal Studies and NBER

Zeuthen Lectures
Copenhagen - October 7-9 2014
The life cycle model: an introduction.

- In economics, we often deal with dynamic problems where individual agents make decisions that have long term consequences.
In economics, we often deal with dynamic problems where individual agents make decisions that have long term consequences.

The obvious example is consumption and saving decisions:
- The choice is between consumption now or consumption later.
- Given a certain amount of total resources...
- ... and some technology for moving resources over time.
In economics, we often deal with dynamic problems where individual agents make decisions that have long term consequences.

The obvious example is consumption and saving decisions:

- The choice is between consumption now or consumption later.
- Given a certain amount of total resources...
- ... and some technology for moving resources over time

Other examples include:

- Labour supply choices.
- Investment in human capital at various points in the life cycle:
  - early years and education;
  - training;
  - health.
- Housing investment.
The life cycle model: an introduction.

- In some cases the choice is intrinsically dynamic:
  - Consumption today v Consumption tomorrow.
The life cycle model: an introduction.

- In some cases the choice is intrinsically dynamic:
  - Consumption today v Consumption tomorrow.

- In some cases additional dynamic elements are introduced by the nature of investment choices:
  - returns to tenure;
  - durables and housing;
  - human capital formation.
In some cases the choice is intrinsically dynamic:
- Consumption today v Consumption tomorrow.

In some cases additional dynamic elements are introduced by the nature of investment choices:
- returns to tenure;
- durables and housing;
- human capital formation.

... or by adjustment costs.
The life cycle model: an introduction.

- In some cases the choice is intrinsically dynamic:
  - Consumption today \( v \) Consumption tomorrow.

- In some cases additional dynamic elements are introduced by the nature of investment choices:
  - returns to tenure;
  - durables and housing;
  - human capital formation.

- ... or by adjustment costs.

- In other cases the dynamic is induced by preferences:
  - Habit formation.
The life cycle model: an introduction.

- The life cycle model constitutes a useful tool which can be used to frame many problems.
The life cycle model constitutes a useful tool which can be used to frame many problems.

The general approach is simple and very powerful:
- Individuals are assumed to live a finite number of periods (which can be random).
- Individuals are assumed to maximise the overall (expected?) utility they receive over the life cycle.
- Individuals are assumed to have certain tools to move resources over time.
The life cycle model: an introduction.

- The life cycle model constitutes a useful tool which can be used to frame many problems.
- The general approach is simple and very powerful:
  - Individuals are assumed to live a finite number of periods (which can be random).
  - Individuals are assumed to maximise the overall (expected?) utility they receive over the life cycle.
  - Individuals are assumed to have certain tools to move resources over time.
- **Footnote:** What is a decision unit: Individuals or households?
Within this framework there are several channels through which dynamics matter:
The life cycle model: an introduction.

Within this framework there are several channels through which dynamics matter:
- Intertemporal trades and moving resources through time.
The life cycle model: an introduction.

- Within this framework there are several channels through which dynamics matter:
  - Intertemporal trades and moving resources through time.
  - Changes in preferences (habits and so on).
The life cycle model: an introduction.

- Within this framework there are several channels through which dynamics matter:
  - Intertemporal trades and moving resources through time.
  - Changes in preferences (habits and so on).
  - Technology of accumulation, transaction costs etc.
The life cycle model: an introduction.

- Within this framework there are several channels through which dynamics matter:
  - Intertemporal trades and moving resources through time.
  - Changes in preferences (habits and so on).
  - Technology of accumulation, transaction costs etc.
- In addition, finite horizon makes the model necessarily non-stationary:
  - This allows us to study interesting phenomena such as:
    - Retirement.
    - Intergenerational dynamics.
    - Evolution of human capital.
The life cycle model: an introduction.

- Within this framework there are several channels through which dynamics matter:
  - Intertemporal trades and moving resources through time.
  - Changes in preferences (habits and so on).
  - Technology of accumulation, transaction costs etc.

- In addition, finite horizon makes the model necessarily non-stationary:
  - This allows us to study interesting phenomena such as:
    - Retirement.
    - Intergenerational dynamics.
    - Evolution of human capital.

- There are some additional features that are key within a life cycle framework
  - Uncertainty: how to formalise it?
  - Discounting: time consistency?
The life cycle model: an introduction.

Within a life cycle perspective, therefore, the essential features of the model are:

- Preferences;
- Technology;
- Intertemporal budget constraints.

How we specify each of these components will define the specific problem we want to address. The level of detail is going to be important: Simple models will be useful to define the relevant tradeoffs and the evidential the main forces at play. Richer models will be necessary to bring the theory to data. The art of modelling: trade-off between simplicity (analytical solutions and intuition) v richness (risk of creating a black box).
The life cycle model: an introduction.

Within a life cycle perspective, therefore, the essential features of the model are:

- Preferences;
- Technology;
- Intertemporal budget constraints.
Within a life cycle perspective, therefore, the essential features of the model are:

- Preferences;
- Technology;
- Intertemporal budget constraints.

How we specify each of these components will define the specific problem we want to address.
The life cycle model: an introduction.

- Within a life cycle perspective, therefore, the essential features of the model are:
  - Preferences;
  - Technology;
  - Intertemporal budget constraints.
- How we specify each of these components will define the specific problem we want to address.
- The level of detail is going to be important:
  - Simple models will be useful to define the relevant tradeoffs and the evidential the main forces at play.
  - Richer models will be necessary to bring the theory to data.
Within a life cycle perspective, therefore, the essential features of the model are:

- Preferences;
- Technology;
- Intertemporal budget constraints.

How we specify each of these components will define the specific problem we want to address.

The level of detail is going to be important:

- Simple models will be useful to define the relevant tradeoffs and the evidential the main forces at play.
- Richer models will be necessary to bring the theory to data.

The art of modelling: trade-off between simplicity (analytical solutions and intuition) v richness (risk of creating a black box).
Outline of Lecture 1: Consumption and labour supply choices.

1. A life cycle approach to individual choices: An introduction.
3. A general model of consumption and labour supply.
4. A simple model with stark restrictions.
5. Characterizing choices.
6. Using the model.
Outline of Lecture 2: Towards less restrictive models.

- Preferences.
  1. Preference heterogeneity in observables and unobservables.
  2. Multiple Commodities
  4. Intertemporal non-separabilities: durability and habits.

- Kinks and Labour supply.
  1. Intensive margins with non separable preferences.
  2. Non convexities and fixed costs: extensive margins.
  3. Returns to experience.

- Market structure and consumption smoothing.
  1. Full risk sharing.
  2. Limited commitment models.
  3. Imperfect information models.
Outline of Lecture 3: The empirics of the life cycle model.

1. Measuring consumption: some serious challenges.
2. Intra-temporal conditions:
   1. Demand Analysis.
   2. Marginal Rates of Substitutions for Labour supply.
3. Approaches based on Euler equations.
   - Aggregation and estimation of preferences.
4. Approaches based on the level of consumption:
   - Calibration and simulation methods.
5. The evolution of second moments of consumption and earnings.
The life cycle model of consumption.

“The fundamental psychological law ... is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income” Keynes (1936).

This sentence was the basis for the static ‘keynesian’ consumption function:

$$C_t = a + bY_t + e_t.$$
“The fundamental psychological law ... is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income” Keynes (1936).

- This sentence was the basis for the static ‘keynesian’ consumption function:

\[ C_t = a + bY_t + e_t. \]

- From a theoretical point of view, such a relationship is very ad-hoc.
The life cycle model of consumption.

“The fundamental psychological law ... is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income” Keynes (1936).

- This sentence was the basis for the static ‘keynesian’ consumption function:

\[ C_t = a + bY_t + e_t. \]

- From a theoretical point of view, such a relationship is very ad-hoc.
- It ignores completely the dynamics, which is surprising as Keynes had discussed with contemporaries much more sophisticated dynamic theories of consumption and saving.
  - Frank Ramsey (1928) Theory of Optimal Saving.
Consumption and saving behaviour: Introduction.

- The Keynesian consumption function, estimated on time series national account data, explains a large fraction of the variance of consumption.
Consumption and saving behaviour: Introduction.

- The Keynesian consumption function, estimated on time series national account data, explains a large fraction of the variance of consumption. 
  but.....

- On macro data, the marginal propensity to consume is lower in the short run than in the long run (Kuznets paradox).

---

**Diagram**: 
A graph showing two lines: Long-Run Consumption Function and Short-Run Consumption Functions.
Consumption and saving behaviour: Introduction.

- On micro data: saving rates change systematically with income.
Consumption and saving behaviour: Introduction.

- On micro data: saving rates change systematically with income.
- For instance: Friedman noted that groups of individuals with, on average, lower level of income (such as blacks) had higher saving rates than other groups with higher levels of average income (such as whites) at any income level.
Consumption and saving behaviour: Introduction.

- On micro data: saving rates change systematically with income.
- For instance: Friedman noted that groups of individuals with, on average, lower level of income (such as blacks) had higher saving rates than other groups with higher levels of average income (such as whites) at any income level.
- Still true to this date. Pooling together US micro data on income and consumption over a number of year (CEX 1980-2003) we find that the median saving rate for blacks is 2.55% higher than for whites, for a given level of real income.
Failures of the Keynesian consumption function.

- Saving rates change systematically with changes in income;
- People whose income has increased save more than people whose income has decreased (Katona, 1949).

\[ i = \alpha + \beta y_i + u_i \]

Modigliani and Brumberg (1954) and Friedman (1957) developed models of intertemporal allocation of consumption that could explain these stylized facts.
Failures of the Keynesian consumption function.

- Saving rates change systematically with changes in income;
- People whose income has increased save more than people whose income has decreased (Katona, 1949).
- Marginal propensity to consume is much larger in time series studies than in microeconomic cross sectional regressions.

\[ c_i = \alpha + \beta y_i + u_i \]
Failures of the Keynesian consumption function.

- Saving rates change systematically with changes in income;
- People whose income has increased save more than people whose income has decreased (Katona, 1949).
- Marginal propensity to consume is much larger in time series studies than in microeconomic cross sectional regressions.

\[ c_i = \alpha + \beta y_i + u_i \]

- Modigliani and Brumberg (1954) and Friedman (1957) developed models of intertemporal allocation of consumption that could explain these stylized facts.
The development of the life-cycle/ permanent income models.

- In both models, consumption is a function of available resources over a long time horizon (life-cycle wealth or permanent income).
The development of the life-cycle/ permanent income models.

- In both models, consumption is a function of available resources over a long time horizon (life-cycle wealth or permanent income).
- Models based on optimization and optimal allocation of resources over time.
  - Analogous to the allocation of total expenditure among different commodities.
  - Current expenditure depends on relative prices (interest rates)...
  - ... and total resources (permanent income).
The development of the life-cycle/ permanent income models.

- In both models, consumption is a function of available resources over a long time horizon (life-cycle wealth or permanent income).
- Models based on optimization and optimal allocation of resources over time.
  - Analogous to the allocation of total expenditure among different commodities.
  - Current expenditure depends on relative prices (interest rates)...
  - ... and total resources (permanent income).
- Interesting relationship between the development of Friedman’s P.I. income model and his interest in measurement error.
  - Study of medical doctors income (Friedman M. and S. Kuznets: "Income from Independent Professional Practice" - NBER 1945).
How the life-cycle/permanent income model works.

- The life cycle/ permanent income model offers a coherent explanation for the stylized facts.
How the life-cycle/permanent income model works.

- The life cycle/permanent income model offers a coherent explanation for the stylized facts.
- Consumers have concave utility functions and therefore prefer smooth paths of consumption (over time and across states of the world) over variable ones.
How the life-cycle/permanent income model works.

- The life cycle/permanent income model offers a coherent explanation for the stylized facts.
- Consumers have concave utility functions and therefore prefer smooth paths of consumption (over time and across states of the world) over variable ones.
- Only unanticipated changes in income that are perceived as permanent induce substantive changes in consumption.
How the life-cycle/permanent income model works.

- The life cycle/permanent income model offers a coherent explanation for the stylized facts.
- Consumers have concave utility functions and therefore prefer smooth paths of consumption (over time and across states of the world) over variable ones.
- Only unanticipated changes in income that are perceived as permanent induce substantive changes in consumption.
- Expected and temporary changes to income should not induce a strong change in consumption.
How the life-cycle/permanent income model works.

- The life cycle/permanent income model offers a coherent explanation for the stylized facts.
- Consumers have concave utility functions and therefore prefer smooth paths of consumption (over time and across states of the world) over variable ones.
- Only unanticipated changes in income that are perceived as permanent induce substantive changes in consumption.
- Expected and temporary changes to income should not induce a strong change in consumption.
- The explanation of many of the stylized facts above boils down to the observation that a large fraction of the changes in income considered are temporary.
How the life-cycle/permanent income model explains the stylized facts.

- Kuznets paradox: short run fluctuations in disposable income are more likely to be dominated by temporary shocks that would be averaged out in the long run.
  - That is why consumption is more responsive to income in the long run than in the short run.
How the life-cycle/permanent income model explains the stylized facts.

- Kuznets paradox: short run fluctuations in disposable income are more likely to be dominated by temporary shocks that would be averaged out in the long run.
  - That is why consumption is more responsive to income in the long run than in the short run.
- The saving rates of blacks is higher than that of whites, at any income level, because the permanent income of blacks is lower and therefore, conditioning on a common income level, one selects the blacks with higher level of temporary shocks.
  - Temporary shocks should, according to the permanent income/life-cycle model, be saved.
How the life-cycle/permanent income model explains the stylized facts.

- Kuznets paradox: short run fluctuations in disposable income are more likely to be dominated by temporary shocks that would be averaged out in the long run.
  - That is why consumption is more responsive to income in the long run than in the short run.
- The saving rates of blacks is higher than that of whites, at any income level, because the permanent income of blacks is lower and therefore, conditioning on a common income level, one selects the blacks with higher level of temporary shocks.
  - Temporary shocks should, according to the permanent income/life-cycle model, be saved.
- Individuals with large income increases are more likely to be affected by positive transitory shocks, that should be saved according to the model.
Perceived empirical failures of the life-cycle/permanent income model.

- Empirical criticisms of the life cycle model have pointed to deviations from the prediction that expected changes in income should not be incorporated into consumption.
Perceived empirical failures of the life-cycle/permanent income model.

- Empirical criticisms of the life cycle model have pointed to deviations from the prediction that expected changes in income should not be incorporated into consumption.
- These deviations can be classified into four groups:
  - correlations between expected changes in income and consumption at low frequencies ("income tracking") (including the "the retirement consumption puzzle");
  - correlations between expected changes in income and consumption at high (business cycle) frequency
  - short run fluctuations linked to ad-hoc payments non necessarily related to labour supply behaviour ("excess sensitivity to transitory receipts").
  - present bias, ‘irrational’ behaviour, and so on.
Does consumption track income over the life cycle?

- Lester Thurow (1969) was one of the first to argue that the fact that consumption tracks income over the life cycle constitutes a rejection of the model.
  - We will look at Heckman (1974) reply.
Does consumption track income over the life cycle?

- Lester Thurow (1969) was one of the first to argue that the fact that consumption tracks income over the life cycle constitutes a rejection of the model.
  - We will look at Heckman (1974) reply.
- Carroll and Summers show that life cycle profiles of income and consumption track each other.
Does consumption track income over the life cycle?

- Lester Thurow (1969) was one of the first to argue that the fact that consumption tracks income over the life cycle constitutes a rejection of the model.
  - We will look at Heckman (1974) reply.
- Carroll and Summers show that life cycle profiles of income and consumption track each other.
- For many countries both income and consumption life cycle profiles are hump shaped, in that they increase during the first part of the life cycle to reach a peak a few years before retirement and decline afterwards.
Does consumption track income over the life cycle?

Lester Thurow (1969) was one of the first to argue that the fact that consumption tracks income over the life cycle constitutes a rejection of the model.

We will look at Heckman (1974) reply.

Carroll and Summers show that life cycle profiles of income and consumption track each other.

For many countries both income and consumption life cycle profiles are hump shaped, in that they increase during the first part of the life cycle to reach a peak a few years before retirement and decline afterwards.

Also groups and countries that exhibit relatively "steep" income profiles also exhibit relatively "steep" consumption profiles;
Does consumption track income over the life cycle?

- We can replicate their findings using UK FES data (1978-2007), where individuals are grouped according to their education attainment.
Does consumption track income over the life cycle?

- We can replicate their findings using UK FES data (1978-2007), where individuals are grouped according to their education attainment.

![Graph showing income and consumption over age for different education levels.]
Does consumption track income over the life cycle?

- We can replicate their findings using UK FES data (1978-2007), where individuals are grouped according to their education attainment.

It is worth distinguishing between total expenditure and expenditure on non durable goods and services.
Consumption Drop at Retirement

- Consumption drops around retirement are documented for the UK (Banks, Blundell and Tanner, 1998), for the US (Bernheim, Skinner and Weinberg, 2001), and for Italy (Battistin, Brugiavini, Rettore and Weber, 2009).
Consumption Drop at Retirement

- Consumption drops around retirement are documented for the UK (Banks, Blundell and Tanner, 1998), for the US (Bernheim, Skinner and Weinberg, 2001), and for Italy (Battistin, Brugiavini, Rettore and Weber, 2009).

- Banks et al (1998) find that there is a cumulated consumption shortfall over the 60-67 age band, where most people retire, of around 10%.
Consumption Drop at Retirement

- Consumption drops around retirement are documented for the UK (Banks, Blundell and Tanner, 1998), for the US (Bernheim, Skinner and Weinberg, 2001), and for Italy (Battistin, Brugiavini, Rettore and Weber, 2009).

- Banks et al (1998) find that there is a cumulated consumption shortfall over the 60-67 age band, where most people retire, of around 10%.

- Bernheim et al (2001) estimate a median drop of 14%, but higher drops for low wealth, low income replacement households.

- Battistin et al (2009) estimate at 9.8 percent the part of the non-durable consumption drop that is associated with retirement (food expenditure falls instead by 14%).
What can explain income tracking?

- Non separability between income and labour supply choices.
What can explain income tracking?

- Non separability between income and labour supply choices.
- Changes in family composition and needs.
What can explain income tracking?

- Non separability between income and labour supply choices.
- Changes in family composition and needs.
- Precautionary savings and income uncertainty.
What can explain income tracking?

- Non separability between income and labour supply choices.
- Changes in family composition and needs.
- Precautionary savings and income uncertainty.
- Borrowing constraints.
What can explain income tracking?

- Non separability between income and labour supply choices.
- Changes in family composition and needs.
- Precautionary savings and income uncertainty.
- Borrowing constraints.
- Changes in life expectancy and discount factors.
What can explain the drop in consumption at retirement?

- Non separability between consumption and leisure.
- Changes in time allocation - Aguiar and Hurst, 2005
- Different definitions of consumption Aguila, Attanasio and Meghir, 2013.
Excess Sensitivity to Predicted Income Changes

- Aggregate consumption correlated to expected income at the business cycle frequency. (Campbell and Mankiw (1989)).
Excess Sensitivity to Predicted Income Changes

- Aggregate consumption correlated to expected income at the business cycle frequency. (Campbell and Mankiw (1989)).

- Consumption seems to react to predictable income changes driven by events that do not affect hours worked or labour force participation, such as tax refunds or other changes linked to administrative issues (Souleles, 1999, Parker, 1999, Hsieh, 2003, Browning and Collado, 2001, and Stephens, 2007).
Excess Sensitivity to Predicted Income Changes

- Aggregate consumption correlated to expected income at the business cycle frequency. (Campbell and Mankiw (1989)).
- Consumption seems to react to predictable income changes driven by events that do not affect hours worked or labour force participation, such as tax refunds or other changes linked to administrative issues (Souleles, 1999, Parker, 1999, Hsieh, 2003, Browning and Collado, 2001, and Stephens, 2007).
  - Souleles, Parker, Stephens and, in part, Hsieh find that consumption reacts to changes in predictable resources available to consumers.
Excess Sensitivity to Predicted Income Changes

- Aggregate consumption correlated to expected income at the business cycle frequency. (Campbell and Mankiw (1989)).
- Consumption seems to react to predictable income changes driven by events that do not affect hours worked or labour force participation, such as tax refunds or other changes linked to administrative issues (Souleles, 1999, Parker, 1999, Hsieh, 2003, Browning and Collado, 2001, and Stephens, 2007).
  - Souleles, Parker, Stephens and, in part, Hsieh find that consumption reacts to changes in predictable resources available to consumers.
  - Browning and Collado and the second part of Hsieh’s paper find that consumers do not respond to such predictable changes in resources.
What can explain excess sensitivity?

- Liquidity constraints and inability to borrow.
  - Juster and Shay (1964).
  - Flemming (1973).
  - Pissarides (1978).

- Precautionary savings and income uncertainty.

- Non separability between income and labour supply choices.

- Aggregation?

- Rules of thumb.
  - Campbell and Mankiw (1989).
What can explain excess sensitivity?

- Liquidity constraints and inability to borrow.
  - Juster and Shay (1964).
  - Flemming (1973).
  - Pissarrides (1978).
- Precautionary savings and income uncertainty.
What can explain excess sensitivity?

- Liquidity constraints and inability to borrow.
  - Juster and Shay (1964).
  - Flemming (1973).
  - Pissarrides (1978).

- Precautionary savings and income uncertainty.

- Non separability between income and labour supply choices.
What can explain excess sensitivity?

- Liquidity constraints and inability to borrow.
  - Juster and Shay (1964).
  - Flemming (1973).
  - Pissarrides (1978).

- Precautionary savings and income uncertainty.

- Non separability between income and labour supply choices.

- Aggregation?
What can explain excess sensitivity?

- Liquidity constraints and inability to borrow.
  - Juster and Shay (1964).
  - Flemming (1973).
  - Pissarrides (1978).

- Precautionary savings and income uncertainty.

- Non separability between income and labour supply choices.

- Aggregation?

- Rules of thumb.
  - Campbell and Mankiw (1989).
Behavioural biases?

- A number of papers have pointed out to deviations from ‘optimal’ behaviour:
  - Present bias, time inconsistencies.
  - Mental accounting.
  - Framing and status quo effects.
- Kaplan and Violante (2011) consider a model with liquid and illiquid assets.
Where does the life cycle model stand?

I will try to argue that the life cycle model constitutes a very useful framework to:

- Put structure on the data;
- Understand certain empirical regularities;
- Inform policies.
Where does the life cycle model stand?

- I will try to argue that the life cycle model constitutes a very useful framework to:
  - Put structure on the data;
  - Understand certain empirical regularities;
  - Inform policies.

- Models that fit the data, however, can be complex and work in ways that are at times surprising.
  - Example of demographics effects on the marginal utility of consumption.
  - Example of saving in recessions (see Alan, Crossley and Low, 2014).
Where does the life cycle model stand?

I will try to argue that the life cycle model constitutes a very useful framework to:

- Put structure on the data;
- Understand certain empirical regularities;
- Inform policies.

Models that fit the data, however, can be complex and work in ways that are at times surprising.

- Example of demographics effects on the marginal utility of consumption.
- Example of saving in recessions (see Alan, Crossley and Low, 2014).

Different versions of the model are useful to analyse different phenomena.
The Life Cycle Model: Theory.

The version of the model we consider is one in which a consumer unit maximizes expected utility over \( T \) periods subject to a set of constraints.

\[
\max_{c, h, s} EU(c, h, z, v) \quad \text{subject to}
\]

\[
W_{t+1} = (W_t + \omega_t h_t - c_t)(1 + R^*_{t+1}).
\]

\[
W_t = \sum_{i=1}^{N} s_{i,t} A^i_t.
\]

\[
R^*_t = \sum_{i=1}^{N} s_{i,t} R^i_t, \quad s_{i,t} = \frac{A^i_t}{W_t}.
\]

\[
W_{T+1} \geq 0.
\]

where \( c = \{c_1, c_2, \ldots, c_T\} \), \( h = \{h_1, h_2, \ldots, h_T\} \), \( z = \{z_1, z_2, \ldots, z_T\} \), \( v = \{v_1, v_2, \ldots, v_T\} \), \( s = \{s_{1,1}, s_{1,2}, \ldots, s_{1,T}, \ldots, s_{N,T}\} \).
This version of the model is relatively general.

Some assumption are already imbedded:

- The household behaves as a unit (Browning, 2000, and Mazzocco, 2007, investigate saving decisions in a collective model).
The Life Cycle Model: Theory.

- This version of the model is relatively general.
- Some assumptions are already imbedded:
  - The household behaves as a unit (Browning, 2000, and Mazzocco, 2007, investigate saving decisions in a collective model).
  - The household maximizes expected utility (Epstein and Zin, 1989, relax this assumption).
This version of the model is relatively general.

Some assumption are already imbedded:

- The household behaves as a unit (Browning, 2000, and Mazzocco, 2007, investigate saving decisions in a collective model).
- The household maximizes expected utility (Epstein and Zin, 1989, relax this assumption).
- Fixed and certain time horizon.
The Life Cycle Model: Theory.

- This version of the model is relatively general.
- Some assumption are already imbedded:
  - The household behaves as a unit (Browning, 2000, and Mazzocco, 2007, investigate saving decisions in a collective model).
  - The household maximizes expected utility (Epstein and Zin, 1989, relax this assumption).
  - Fixed and certain time horizon.
  - No fixed costs of going to work.
The Life Cycle Model: Theory.

- This version of the model is relatively general.
- Some assumption are already imbedded:
  - The household behaves as a unit (Browning, 2000, and Mazzocco, 2007, investigate saving decisions in a collective model).
  - The household maximizes expected utility (Epstein and Zin, 1989, relax this assumption).
  - Fixed and certain time horizon.
  - No fixed costs of going to work.
  - No bequest motive.
The Life Cycle Model: Theory.

- This version of the model is relatively general.
- Some assumption are already imbedded:
  - The household behaves as a unit (Browning, 2000, and Mazzocco, 2007, investigate saving decisions in a collective model).
  - The household maximizes expected utility (Epstein and Zin, 1989, relax this assumption).
  - Fixed and certain time horizon.
  - No fixed costs of going to work.
  - No bequest motive.
  - No limits to positions in given assets and rates of return independent of the position.
The Life Cycle Model: Theory.

- This characterisation is useful in that it makes it clear the main components of the model:
  1. Preferences: utility $U$ and assumption about expected utility
  2. Resources: wages!
The Life Cycle Model: Theory.

- It is however useful to restrict the model further to better understand its working.
- Many of these assumptions will be relaxed.
The Life Cycle Model: Theory.

- It is however useful to restrict the model further to better understand its working.
- Many of these assumptions will be relaxed.
- So we will assume:
  - Intertemporal separability
  - Future is discounted by a (time varying) factor $\beta_{t+j}$
    - It may be reflecting survival probabilities.
    - or changing needs (family composition, age etc.)
The Life Cycle Model: Theory.

- It is however useful to restrict the model further to better understand its working.
- Many of these assumptions will be relaxed.
- So we will assume:
  - Intertemporal separability
  - Future is discounted by a (time varying) factor $\beta_{t+j}$
    - It may be reflecting survival probabilities.
    - or changing needs (family composition, age etc.)
  - A single asset $W$.
    - we are ignoring for the time being portfolio choices
The Life Cycle Model: Theory.

\[ \max_{c,h} E_t \sum_{t=1}^{T} \beta_t U(c_t, h_t, z_t, v_t) \] subject to

\[ W_{t+1} = (W_t + \omega_t h_t - c_t)(1 + R_t^{*}) \]

\[ W_{T+1} \geq 0. \]
The Life Cycle Model: Notation and assumptions.

- The observable taste shifters variables, $z_t$, can be a choice variable or not.

Wages $\omega_t$, interest rates $R_t$ and possibly the $z_t$ and $v_t$ variables are uncertain. Wages and interest rates are considered as exogenous processes that define the environment individuals face. The returns on the single asset do not depend on the position taken by the household.
The observable taste shifters variables, $z_t$, can be a choice variable or not.

There are unobservable factors, $v_t$, that also affect utility.
The observable taste shifters variables, \( z_t \), can be a choice variable or not.

There are unobservable factors, \( v_t \), that also affect utility.

Wages \( \omega_t \), interest rates \( R_t \) and possibly the \( z_t \) and \( v_t \) variables are uncertain.
The observable taste shifters variables, $z_t$, can be a choice variable or not.

There are unobservable factors, $v_t$, that also affect utility.

Wages $\omega_t$, interest rates $R_t$ and possibly the $z_t$ and $v_t$ variables are uncertain.

Wages and interest rates are considered as exogenous processes that define the environment individuals face.
The observable taste shifters variables, $z_t$, can be a choice variable or not.

There are unobservable factors, $v_t$, that also affect utility.

Wages $\omega_t$, interest rates $R_t$ and possibly the $z_t$ and $v_t$ variables are uncertain.

Wages and interest rates are considered as exogenous processes that define the environment individuals face.

The returns on the single asset do not depend on the position taken by the household.
The Life Cycle Model: Consumption and Labour Supply Choices.

Given the assumptions made so far, the problem can be analysed with the help of dynamic programming tools.
Given the assumptions made so far, the problem can be analysed with the help of dynamic programming tools.

Notice that in this problem, (if the variables $\omega_t, z_t, R_t, v_t$ are i.i.d., the only dynamics is induced by the intertemporal budget constraint.)
Given the assumptions made so far, the problem can be analysed with the help of dynamic programming tools.

Notice that in this problem, (if the variables $\omega_t$, $z_t$, $R_t$, $v_t$ are i.i.d.,) the only dynamics is induced by the intertemporal budget constraint.

Additional dynamics could arise:

- from the stochastic processes that govern the exogenous variables: wages, interest rates and, possibly, the taste shifters;
- from preferences;
- from durables, transaction costs etc.
The Life Cycle Model: Consumption and Labour Supply Choices.

- If the exogenous variables are i.i.d. the problem can be characterised as a function of a single state variable.
The Life Cycle Model: Consumption and Labour Supply Choices.

- If the exogenous variables are i.i.d. the problem can be characterised as a function of a single state variable.
- We can characterise the optimal choices by looking at:
  - a set of Euler equations that govern the dynamics of the problem
  - Marginal Rates of Substitutions that govern the within period allocation of resources (hours and consumption).
The Life Cycle Model: Consumption and Labour Supply Choices.

- Euler equation:

\[ \lambda_t = E_t[\lambda_{t+1}(1 + R_{t+1}^*)] \]  

where \( \lambda_t \) is the Lagrange multiplier associated to the intertemporal budget constraint between \( t \) and \( t + 1 \).
The Life Cycle Model: Consumption and Labour Supply Choices.

- **Euler equation:**
  \[ \lambda_t = E_t[\lambda_{t+1}(1 + R^*_{t+1})] \]  
  where \( \lambda_t \) is the Lagrange multiplier associated to the intertemporal budget constraint between \( t \) and \( t + 1 \).

- **The intratemporal first order condition:**
  \[ U_{ct} = U_{ht} \omega_t \]
The Life Cycle Model: Consumption and Labour Supply Choices.

- Euler equation:
  \[ \lambda_t = E_t[\lambda_{t+1}(1 + R_{t+1}^*)] \]  \hspace{1cm} (1)
  where \( \lambda_t \) is the Lagrange multiplier associated to the intertemporal budget constraint between \( t \) and \( t + 1 \).

- The intratemporal first order condition:
  \[ U_{ct} = U_{ht} \omega_t \]

- We also have:
  \[ \beta_t U_{ct} = \lambda_t \]  \hspace{1cm} (2)
The Life Cycle Model: Consumption and Labour Supply Choices.

- Euler equation:
  \[ \lambda_t = E_t[\lambda_{t+1}(1 + R_{t+1}^*)] \] (1)
  where \( \lambda_t \) is the Lagrange multiplier associated to the intertemporal budget constraint between \( t \) and \( t + 1 \).

- The intratemporal first order condition:
  \[ U_{ct} = U_{ht} \omega_t \]

- We also have:
  \[ \beta_t U_{ct} = \lambda_t \] (2)

- Combining equations (1) and (2) we get the Euler equation for consumption:
  \[ \beta_t U_{ct} = E_t[\beta_{t+1}U_{ct+1}(1 + R_{t+1}^*)] \]
The Euler equation for consumption

\[ \beta_t U_{c_t} = E_t[\beta_{t+1} U_{c_{t+1}} (1 + R_{t+1}^*)] \]

- This equation gives us the essence of the life cycle model for consumption
The Euler equation for consumption

\[ \beta_t U_{c_t} = E_t[\beta_{t+1} U_{c_{t+1}} (1 + R_{t+1}^*)] \]

- This equation gives us the essence of the life cycle model for consumption.
- It says that the appropriately discounted marginal utility of consumption is kept constant in expectation.
The Euler equation for consumption

\[ \beta_t U_c = E_t[\beta_{t+1} U_{c_{t+1}} (1 + R_{t+1}^*)] \]

- This equation gives us the essence of the life cycle model for consumption.
- It says that the appropriately discounted marginal utility of consumption is kept constant in expectation.
- Notice that the marginal utility will depend on \( h_t, z_t \) and \( v_t \).
The Euler equation for consumption

\[ \beta_t U_{ct} = E_t[\beta_{t+1} U_{ct+1} (1 + R_{t+1}^*)] \]

- This equation gives us the essence of the life cycle model for consumption.
- It says that the appropriately discounted marginal utility of consumption is kept constant in expectation.
- Notice that the marginal utility will depend on \( h_t, z_t \) and \( v_t \).
- An analogous equation could be considered for labour supply.

\[ \beta_t U_{ht} = E_t[\beta_{t+1} \frac{\omega_t}{\omega_{t+1}} U_{ht+1} (1 + R_{t+1}^*)] \]
The Euler equation for consumption

- The Euler equations, the terminal condition and the intertemporal budget constraint fully characterize the solution.
The Euler equation for consumption

- The Euler equations, the terminal condition and the intertemporal budget constraint fully characterize the solution.
- In general, it is impossible to derive a closed form solution for consumption.
The Euler equation for consumption

- The Euler equations, the terminal condition and the intertemporal budget constraint fully characterize the solution.
- In general, it is impossible to derive a closed form solution for consumption.
- The Euler equation is an equilibrium condition that can be used to characterise some properties of the solution.
  - Hall (1978), for instance, noticed that, conditional on time $t$ consumption, other variables known at time $t$ should not predict (the marginal utility) of time $t + 1$ consumption.
The Euler equation for consumption

- The Euler equations, the terminal condition and the intertemporal budget constraint fully characterize the solution.

- In general, it is impossible to derive a closed form solution for consumption.

- The Euler equation is an equilibrium condition that can be used to characterise some properties of the solution.
  - Hall (1978), for instance, noticed that, conditional on time $t$ consumption, other variables known at time $t$ should not predict (the marginal utility) of time $t + 1$ consumption.

- The Euler equation, however, is not a consumption function.
The Euler equation for consumption

- The Euler equations, the terminal condition and the intertemporal budget constraint fully characterize the solution.
- In general, it is impossible to derive a closed form solution for consumption.
- The Euler equation is an equilibrium condition that can be used to characterise some properties of the solution.
  - Hall (1978), for instance, noticed that, conditional on time $t$ consumption, other variables known at time $t$ should not predict (the marginal utility) of time $t + 1$ consumption.
- The Euler equation, however, is *not* a consumption function.
- To obtain a consumption function we need to solve the problem completely, either numerically or analytically.
- Analytical solution are only possible under further restrictions.
The Life Cycle Model: facts v theory.

- Flexible versions of this model can explain all three the stylised facts discussed above.
The Life Cycle Model: facts v theory.

- Flexible versions of this model can explain all three the stylised facts discussed above.
- ‘Income tracking’ and the hump in the age profile of consumption is due to the interplay of demographics and prudence.
  - Attanasio, Banks, Meghir and Weber (1999): ”Humps and bumps in life time consumption”.
  - Differences in discount factor explain difference in shapes across education groups.
The Life Cycle Model: facts v theory.

- Flexible versions of this model can explain all three the stylised facts discussed above.
- ‘Income tracking’ and the hump in the age profile of consumption is due to the interplay of demographics and prudence.
  - Attanasio, Banks, Meghir and Weber (1999): ”Humps and bumps in life time consumption”.
  - Differences in discount factor explain difference in shapes across education groups.
- The excess sensitivity of consumption growth to income growth can be due to the dependence of the marginal utility of consumption on leisure.
The Life Cycle Model: facts v theory.

- Flexible versions of this model can explain all three the stylised facts discussed above.
- ‘Income tracking’ and the hump in the age profile of consumption is due to the interplay of demographics and prudence.
  - Attanasio, Banks, Meghir and Weber (1999): ”Humps and bumps in life time consumption”.
  - Differences in discount factor explain difference in shapes across education groups.
- The excess sensitivity of consumption growth to income growth can be due to the dependence of the marginal utility of consumption on leisure.
- The retirement consumption drop is due partly to adverse shocks inducing retirement, partly to more efficient shopping that is made convenient by the increased leisure time.
The Life Cycle Model: facts v theory.

But:
- Relatively simple version of the model do not fit older and younger ages:
  - Are the young liquidity constrained?
  - Does health change dramatically in the last part of the life cycle?
The Life Cycle Model: facts v theory.

But:

- Relatively simple version of the model do not fit older and younger ages:
  - Are the young liquidity constrained?
  - Does health change dramatically in the last part of the life cycle?
- To explain excess sensitivity to some ad-hoc income changes one may have to appeal to different phenomena:
  - Liquidity constraints.
  - Liquidity (Kaplan and Violante, 2014).
  - Non standard preferences, mental accounting.
To check what are the properties of the model we need to characterize the solution to the optimization problem.
The Life Cycle Model: facts v theory.

- To check what are the properties of the model we need to characterize the solution to the optimization problem.
- Some features of the solution can be understood by looking at the first order conditions;
- Others require the derivation of the consumption function, either analytically (in some special cases) or numerically.
Separability between consumption and leisure

- When can income \( (y_t = h_t \omega_t) \) be considered as an exogenous variable?
Separability between consumption and leisure

When can *income* \( y_t = h_t \omega_t \) be considered as an exogenous variable?

- Additive separability between leisure and consumption in the utility function.
- The problem can then be seen as *conditional* on optimal labour supply choices.
Separability between consumption and leisure

- When can income \((y_t = h_t \omega_t)\) be considered as an exogenous variable?
  - Additive separability between leisure and consumption in the utility function.
  - The problem can then be seen as *conditional* on optimal labour supply choices.
- If we make the assumption of additively separable leisure and consumption we can treat labour supply as coming from the solution of the appropriate problem and treat income as an exogenous stochastic process.
Separability between consumption and leisure

- We can then re-write our problem as:

$$\max_{c, h} E_t \sum_{t=1}^{T} \beta_t U(c_t, h_t, z_t, v_t) \quad \text{subject to}$$

$$W_{t+1} = (W_t + y_t - c_t)(1 + R_{t+1}^*)$$

$$W_{T+1} \geq 0.$$

where $y_t = h_t^* \omega_t$ and $h_t^*$ is the equilibrium ‘optimal’ level of labour supply.
Separability between consumption and leisure

- We can then re-write our problem as:

\[
\max_{c, h} E_t \sum_{t=1}^{T} \beta_t U(c_t, h_t, z_t, v_t) \quad \text{subject to}
\]

\[
W_{t+1} = (W_t + y_t - c_t)(1 + R_{t+1}^*)
\]

\[
W_{T+1} \geq 0.
\]

where \( y_t = h_t^* \omega_t \) and \( h_t^* \) is the equilibrium ‘optimal’ level of labour supply.

- In this problem it is useful to define a variable, sometimes denoted as ‘cash-in-hand’

\[
X_t = W_t + y_t.
\]

Notice that:

\[
X_{t+1} = (X_t - c_t)(1 + R_{t+1}^*) + y_{t+1}.
\]
Let's consider the value function for the maximization problem faced by the consumer.
Let's consider the value function for the maximization problem faced by the consumer.

We will consider the case where only income and interest rates are random variable and are i.i.d.
The Life Cycle Model: Characterizing the solution.

- Let's consider the value function for the maximization problem faced by the consumer.
- We will consider the case where only income and interest rates are random variable and are i.i.d. .
- For simplicity we will ignore the portfolio problem and consider a single interest rate.
The Life Cycle Model: Characterizing the solution.

- Let’s consider the value function for the maximization problem faced by the consumer.
- We will consider the case where only income and interest rates are random variable and are i.i.d.
- For simplicity we will ignore the portfolio problem and consider a single interest rate.
- For simplicity we also consider the case with no $z$ or $v$ variables and with geometric discounting.
The Life Cycle Model: Characterizing the solution.

- Let’s consider the value function for the maximization problem faced by the consumer.
- We will consider the case where only income and interest rates are random variable and are i.i.d.
- For simplicity we will ignore the portfolio problem and consider a single interest rate.
- For simplicity we also consider the case with no \( z \) or \( v \) variables and with geometric discounting.
- In this case the only state variable is cash in hand \( X = W + y \).
The value function at the beginning of time is given by:

\[ V(X_0) = \max_{E_0} \sum_{t=0}^{T} \beta^t U(c_t), \quad \text{subject to} \]

\[ X_{t+1} = (X_t - c_t)(1 + R_{t+1}^*) + y_{t+1} \]

\[ X_t = W_t + y_t \]

\[ V(X_{T+1}) = 0 \]
The value function at the beginning of time is given by:

\[ V(X_0) = \max E_0 \sum_{t=0}^{T} \beta^t U(c_t), \quad \text{subject to} \]

\[ X_{t+1} = (X_t - c_t)(1 + R^*_t) + y_t + 1 \]

\[ X_t = W_t + y_t \]

\[ V(X_{T+1}) = 0 \]

The corresponding Bellman equation is given by:

\[ V(X_t) = \max_{c_t} \{ U(c_t) + \beta E_t[V((X_t - c_t)(1 + R^*_t) + y_t + 1)] \} \]

the \( E_t \) operator is conditional on information available at \( t \).
The Life Cycle Model: Characterizing the solution.

The value function at the beginning of time is given by:

\[ V(X_0) = \max E_0 \sum_{t=0}^{T} \beta^t U(c_t), \quad \text{subject to} \]

\[ X_{t+1} = (X_t - c_t)(1 + R^*_t) + y_{t+1} \]
\[ X_t = W_t + y_t \]
\[ V(X_{T+1}) = 0 \]

The corresponding Bellman equation is given by:

\[ V(X_t) = \max_{c_t} \{ U(c_t) + \beta E_t[V((X_t - c_t)(1 + R^*_t) + y_{t+1})] \} \]

the \( E_t \) operator is conditional on information available at \( t \).

In this case, the Kuhn-Tucker multiplier associated with the intertemporal budget constraint at time \( t \) is: \( \lambda_t = \frac{\partial V(X_t)}{\partial X_t} \)
The Life Cycle Model: Characterizing the solution.

- The Euler equation for this problem is identical to the one we saw already.

\[
\frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta (1 + R_{t+1}^*) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right]
\]
The Life Cycle Model: Characterizing the solution.

- The Euler equation for this problem is identical to the one we saw already.

\[
\frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta (1 + R^*_{t+1}) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right]
\]

- To get an analytical solution we will need additional assumptions.
- The main difficult arises from the explicit consideration of uncertainty.
The Life Cycle Model: Characterizing the solution.

- The Euler equation for this problem is identical to the one we saw already.

\[
\frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta (1 + R^*_t) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right]
\]

- To get an analytical solution we will need additional assumptions.
- The main difficult arises from the explicit consideration of uncertainty.
- If assets or income had persistence one would have additional state variables as the current state of a persistent random variable would give information on the future value of the problem over and above the value of cash in hand.
The Life Cycle Model: Characterizing the solution.

- The Euler equation for this problem is identical to the one we saw already.

\[
\frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta (1 + R^*_t) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right]
\]

- To get an analytical solution we will need additional assumptions.
- The main difficult arises from the explicit consideration of uncertainty.
- If assets or income had persistence one would have additional state variables as the current state of a persistent random variable would give information on the future value of the problem over and above the value of cash in hand.
- As it is the only intertemporal link in this problem is given by cash in hand.
The Life Cycle Model: Characterizing the solution.

- The discounted marginal utility of consumption is kept constant in expected value.
The discounted marginal utility of consumption is kept constant in expected value.

The marginal utility of wealth summarize all the information we need to compute consumption (including the information about future variables).
The Life Cycle Model: Characterizing the solution.

- The discounted marginal utility of consumption is kept constant in expected value.
- The marginal utility of wealth summarize all the information we need to compute consumption (including the information about future variables).
- At the optimum the marginal utility of wealth evolves like a martingale.
Closed form solutions under quadratic preferences.

Let’s assume that utility is quadratic:

$$U(c_t) = c_t - bc_t^2, \quad b > 0, \quad 0 < c_t \leq 1/2b.$$
Closed form solutions under quadratic preferences.

- Let’s assume that utility is quadratic:

\[ U(c_t) = c_t - bc_t^2, \quad b > 0, \quad 0 < c_t \leq 1/2b. \]

- The limit on the support is necessary to ensure \( U' > 0 \).
Closed form solutions under quadratic preferences.

Let’s assume that utility is quadratic:

\[ U(c_t) = c_t - bc_t^2, \quad b > 0, \quad 0 < c_t \leq 1/2b. \]

The limit on the support is necessary to ensure \( U' > 0 \).

Quadratic utility implies linear marginal utility.
Closed form solutions under quadratic preferences.

- Let’s assume that utility is quadratic:
  \[ U(c_t) = c_t - bc_t^2, \quad b > 0, \quad 0 < c_t \leq 1/2b. \]

- The limit on the support is necessary to ensure \( U' > 0 \).
- Quadratic utility implies linear marginal utility.
- We also assume that the interest rate is a constant \( R \) and that \( \beta(1 + R) = 1 \).
Closed form solutions under quadratic preferences.

- Let’s assume that utility is quadratic:
  \[ U(c_t) = c_t - bc_t^2, \quad b > 0, \quad 0 < c_t \leq 1/2b. \]

- The limit on the support is necessary to ensure \( U' > 0 \).
- Quadratic utility implies linear marginal utility.
- We also assume that the interest rate is a constant \( R \) and that \( \beta(1 + R) = 1 \).
- The only stochastic variable is income \( y_t \).
Closed form solutions under quadratic preferences.

- Under these conditions the Euler equation will be:

\[ c_t = E_t c_{t+1} \]
Closed form solutions under quadratic preferences.

- Under these conditions the Euler equation will be:

\[ c_t = E_t c_{t+1} \]

- We can rewrite this as:

\[ c_{t+1} = c_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \text{ such that } E_t \varepsilon_{t+1} = 0. \]
Closed form solutions under quadratic preferences.

- Under these conditions the Euler equation will be:
  \[ c_t = E_t c_{t+1} \]

- We can rewrite this as:
  \[ c_{t+1} = c_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \text{ such that } E_t \epsilon_{t+1} = 0. \]

- Using the Euler equation and the intertemporal budget constraint one can derive the consumption function:
  \[ \rho_t c_t = \frac{R}{1 + R} A_t + \frac{R}{1 + R} \sum_{j=0}^{T-t} \frac{1}{(1 + R)^j} E_t [y_{t+j}] \]
  where \( \rho_t = 1 - \frac{1}{(1+R)^{T-t+1}}, \quad \lim_{t \to \infty} \rho_t = 1 \)
Closed form solutions under quadratic preferences.

- The Euler equation highlights the consumption smoothing properties of the solution, emphasized by Modigliani and Brumberg (1954).
Closed form solutions under quadratic preferences.

- The Euler equation highlights the consumption smoothing properties of the solution, emphasized by Modigliani and Brumberg (1954).
- The consumption function makes clear the other main implication of the model, first stressed in Friedman (1957): consumption depends on the present discounted value of future expected income.
Closed form solutions under quadratic preferences.

- The Euler equation highlights the consumption smoothing properties of the solution, emphasized by Modigliani and Brumberg (1954).
- The consumption function makes clear the other main implication of the model, first stressed in Friedman (1957): consumption depends on the present discounted value of future expected income.
- The interest rate plays the role of converting future resources to present ones and therefore constitutes an important determinant of consumption: it is a relative price.
Closed form solutions under quadratic preferences.

- The Euler equation highlights the consumption smoothing properties of the solution, emphasized by Modigliani and Brumberg (1954).
- The consumption function makes clear the other main implication of the model, first stressed in Friedman (1957): consumption depends on the present discounted value of future expected income.
- The interest rate plays the role of converting future resources to present ones and therefore constitutes an important determinant of consumption: it is a relative price.
- Appraising the effects of a given policy, for instance a tax reform that affects disposable income, a distinction must be drawn between permanent and temporary changes (Blinder and Deaton, 1985, Poterba, 1988).
Closed form solutions under quadratic preferences.

- The reason why consumption is 'on average' constant is because $\beta(1 + R) = 1$. 

If the interest rate is larger than the discount rate, consumption will be increasing. If the interest rate is smaller than the discount rate (people are relatively impatient), consumption will be decreasing. When the discount rate changes with age (maybe because of mortality risk or because of changing needs), the consumption profile might be increasing over some intervals and decreasing over others.

The shape of the consumption age profile is independent of the properties of income. This follows from:

- The assumption that individuals can borrow and lend freely at interest rate $R$.
- The assumption that income process is exogenous. (separability between leisure and consumption in the utility function).
Closed form solutions under quadratic preferences.

- The reason why consumption is 'on average' constant is because $\beta(1 + R) = 1$.
  - If the interest rate is larger than the discount rate, consumption will be increasing.
  - If the interest rate is smaller than the discount rate (people are relatively impatient), consumption will be decreasing.
Closed form solutions under quadratic preferences.

- The reason why consumption is 'on average' constant is because $\beta (1 + R) = 1$.
  - If the interest rate is larger than the discount rate consumption will be increasing.
  - If the interest rate is smaller than the discount rate (people are relatively impatient), consumption will be decreasing.
  - When the discount rate changes with age (maybe because of mortality risk or because of changing needs), the consumption profile might be increasing over some intervals and decreasing over others.
Closed form solutions under quadratic preferences.

- The reason why consumption is 'on average' constant is because $\beta(1 + R) = 1$.
  - If the interest rate is larger than the discount rate consumption will be increasing.
  - If the interest rate is smaller than the discount rate (people are relatively impatient), consumption will be decreasing.
  - When the discount rate changes with age (maybe because of mortality risk or because of changing needs), the consumption profile might be increasing over some intervals and decreasing over others.
- The shape of the consumption age profile is independent of the properties of income. This follows from:
  - The assumption that individuals can borrow and lend freely at interest rate $R$. 

Orazio Attanasio (UCL & IFS)
Closed form solutions under quadratic preferences.

- The reason why consumption is ‘on average’ constant is because \( \beta(1 + R) = 1 \).
  - If the interest rate is larger than the discount rate consumption will be increasing.
  - If the interest rate is smaller than the discount rate (people are relatively impatient), consumption will be decreasing.
  - When the discount rate changes with age (maybe because of mortality risk or because of changing needs), the consumption profile might be increasing over some intervals and decreasing over others.

- The shape of the consumption age profile is independent of the properties of income. This follows from:
  - The assumption that individuals can borrow and lend freely at interest rate \( R \).
  - The assumption that income process is exogenous. (separability between leisure and consumption in the utility function).
Closed form solutions under quadratic preferences.

From the solution for consumption we can derive an interesting expression.
Closed form solutions under quadratic preferences.

From the solution for consumption we can derive an interesting expression. Let's consider for notational simplicity the infinite horizon case, so that $\rho_t = 1$. 
Closed form solutions under quadratic preferences.

From the solution for consumption we can derive an interesting expression. Let’s consider for notational simplicity the infinite horizon case, so that $\rho_t = 1$.

Notice that the Euler equation implies:

$$E_t[c_{t+1} - c_t] = 0, \quad \text{or} \quad c_{t+1} - c_t = \epsilon_{t+1}$$
Closed form solutions under quadratic preferences.

From the solution for consumption we can derive an interesting expression. Let’s consider for notational simplicity the infinite horizon case, so that \( \rho_t = 1 \).

Notice that the Euler equation implies:

\[
E_t [c_{t+1} - c_t] = 0, \quad \text{or} \quad c_{t+1} - c_t = \epsilon_{t+1}
\]

\[
c_t = \frac{R}{1+R} A_t + \frac{R}{1+R} E_t \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+j} \right]
\]
Closed form solutions under quadratic preferences.

From the solution for consumption we can derive an interesting expression. Let's consider for notational simplicity the infinite horizon case, so that $\rho_t = 1$. Notice that the Euler equation implies:

$$E_t[c_{t+1} - c_t] = 0,$$

or

$$c_{t+1} - c_t = \epsilon_{t+1}$$

$$c_t = \frac{R}{1+R} A_t + \frac{R}{1+R} E_t \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+j} \right]$$

$$c_{t+1} = \frac{R}{1+R} A_{t+1} + \frac{R}{1+R} E_{t+1} \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right]$$
Closed form solutions under quadratic preferences.

From the solution for consumption we can derive an interesting expression. Let's consider for notational simplicity the infinite horizon case, so that $\rho_t = 1$. Notice that the Euler equation implies:

$$E_t[c_{t+1} - c_t] = 0, \quad \text{or} \quad c_{t+1} - c_t = \epsilon_{t+1}$$

$$c_t = \frac{R}{1+R} A_t + \frac{R}{1+R} E_t \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+j} \right]$$

$$c_{t+1} = \frac{R}{1+R} A_{t+1} + \frac{R}{1+R} E_{t+1} \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right]$$

$$= \frac{R}{1+R} (1 + R) (A_t + y_t - c_t) + \frac{R}{1+R} E_{t+1} \left[ \sum_{j=1}^{\infty} \frac{1}{(1+R)^{j-1}} y_{t+j} \right]$$
Closed form solutions under quadratic preferences.

Appropriate manipulation yields:

\[
    c_{t+1} = c_t + \frac{R}{1+R} E_{t+1} \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right]
    - \frac{R}{1+R} E_t \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right]
\]
Closed form solutions under quadratic preferences.

Appropriate manipulation yields:

\[ c_{t+1} = c_t + \frac{R}{1+R} E_{t+1} \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right] - \frac{R}{1+R} E_t \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right] \]

Suppose that the income process is given by:

\[ A(L)y_{t+1} = a + \zeta_{t+1} \]

Then the equation above implies:

\[ A\left(\frac{1}{1+R}\right) \Delta c_{t+1} = \frac{R}{1+R} \zeta_{t+1} \]
Closed form solutions under quadratic preferences.

Appropriate manipulation yields:

\[ c_{t+1} = c_t + \frac{R}{1+R} E_{t+1} \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right] \]

\[ -\frac{R}{1+R} E_t \left[ \sum_{j=0}^{\infty} \frac{1}{(1+R)^j} y_{t+1+j} \right] \]

Suppose that the income process is given by:

\[ A(L)y_{t+1} = a + \zeta_{t+1} \]

Then the equation above implies:

\[ A \left( \frac{1}{1+R} \right) \Delta c_{t+1} = \frac{R}{1+R} \zeta_{t+1} \]

- Notice that we have two time series representations (one for income and one for consumption) that imply cross equation restrictions.
- These can be tested (Flavin (1981)).
Cross equation restrictions linking consumption and income processes.

- Flavin’s restrictions were reinterpreted by Campbell (1987): Saving for a rainy day.
- Consumption should decrease in anticipation of declines in income.
Cross equation restrictions linking consumption and income processes.

- Flavin’s restrictions were reinterpreted by Campbell (1987): Saving for a rainy day.
- Consumption should decrease in anticipation of declines in income.
- If there is positive serial correlation in the first differenced process of income, then consumption growth should vary more than income growth over time.
Cross equation restrictions linking consumption and income processes.

- Flavin’s restrictions were reinterpreted by Campbell (1987): Saving for a rainy day.
- Consumption should decrease in anticipation of declines in income.
- If there is positive serial correlation in the first differenced process of income, then consumption growth should vary more than income growth over time.
- Campbell and Deaton (1989) and West (1988) use the same structure to propose a test that links the innovation to permanent income to consumption.
Cross equation restrictions linking consumption and income processes.

- Flavin’s restrictions were reinterpreted by Campbell (1987) : Saving for a rainy day.
- Consumption should decrease in anticipation of declines in income.
- If there is positive serial correlation in the first differenced process of income, then consumption growth should vary more than income growth over time.
- Campbell and Deaton (1989) and West (1988) use the same structure to propose a test that links the innovation to permanent income to consumption.
- Campbell and Deaton (1989) present evidence that aggregate consumption is "excessively smooth" in that it does not react enough to news about income.
Cross sectional second moments..

- Once again notice that

\[ c_{t+1}^h = c_t^h + e_{t+1}^h, \quad E_t[e_{t+1}^h] = 0 \]

where the superscript \( h \) denotes households.
Cross sectional second moments..

- Once again notice that

$$c^h_{t+1} = c^h_t + e^h_{t+1}, \quad E_t[e^h_{t+1}] = 0$$

where the superscript $h$ denotes households.

- $c^h_{t+1}$ is therefore a martingale, for each individual household.
Once again notice that

\[ c_{t+1}^h = c_t^h + e_{t+1}^h, \quad E_t[e_{t+1}^h] = 0 \]

where the superscript \( h \) denotes households.

- \( c_{t+1}^h \) is therefore a martingale, for each individual household.
- The innovations to the martingale are innovations to (permanent) income.
Cross sectional second moments..

- Once again notice that

\[ c_{t+1}^h = c_t^h + e_{t+1}^h, \quad E_t[e_{t+1}^h] = 0 \]

where the superscript \( h \) denotes households.

- \( c_{t+1}^h \) is therefore a martingale, for each individual household.

- The innovations to the martingale are innovations to (permanent) income.

- Notice that the cross sectional variance of consumption will be increasing over time;
  - unless innovation are not perfectly correlated across households.
  - See Deaton and Paxon (1994).
Notice also that consumption at time \( t \) will be the sum of \( t \) independent shocks.

- If \( t \) is sufficiently large, by a central limit theorem this will be normally distributed, even if the original shocks are not.
- See Battistin, Blundell and Lewbell (2009).
Central limit theorem for the consumption distribution?.

- Notice also that consumption at time $t$ will be the sum of $t$ independent shocks.
  - If $t$ is sufficiently large, by a central limit theorem this will be normally distributed, even if the original shocks are not.
  - See Battistin, Blundell and Lewbell (2009).
  - This result will change a bit if one deviates from quadratic utility.
Quadratic utility makes the algebra easy because the marginal utility is linear.

In general it is not possible to derive closed form solutions for consumption.
Quadratic utility makes the algebra easy because the marginal utility is linear.

In general it is not possible to derive closed form solutions for consumption.

The linearity of the marginal utility implies that people behave as if means are substituted for actual values: \( \rightarrow \) ‘certainty equivalence’.

Zeuthen Lecture 1 : 7/10/14
The Life Cycle Model: Precautionary savings.

- Quadratic utility makes the algebra easy because the marginal utility is linear.
- In general it is not possible to derive closed form solutions for consumption.
- The linearity of the marginal utility implies that people behave as if means are substituted for actual values: → 'certainty equivalence'.
- But it implies that changes in the variability (risk) of income processes or interest rate processes has no effect on consumption.
The Life Cycle Model: Precautionary savings.

- Quadratic utility makes the algebra easy because the marginal utility is linear.
- In general it is not possible to derive closed form solutions for consumption.
- The linearity of the marginal utility implies that people behave as if means are substituted for actual values: \( \rightarrow \) ’certainty equivalence’.
- But it implies that changes in the variability (risk) of income processes or interest rate processes has no effect on consumption.
- Alternative formulations allow for ’precautionary saving’:
  - A reduction in current consumption in the face of additional risk.
The Life Cycle Model: Precautionary savings.

- Quadratic utility makes the algebra easy because the marginal utility is linear.
- In general it is not possible to derive closed form solutions for consumption.
- The linearity of the marginal utility implies that people behave as if means are substituted for actual values: → 'certainty equivalence'.
- But it implies that changes in the variability (risk) of income processes or interest rate processes has no effect on consumption.
- Alternative formulations allow for 'precautionary saving':
  - A reduction in current consumption in the face of additional risk.
- Dreze and Modigliani (1972): the properties of the third derivative of the utility function (or on the convexity of marginal utility) are key.
The Life Cycle Model: Precautionary savings.

A case when the marginal utility is not linear and one can still obtain a closed form solution is when the utility function exhibits constant absolute risk aversion.

\[ U(c_t) = -\frac{1}{k} \exp\{-kc_t\} \]
The Life Cycle Model: Precautionary savings.

- A case when the marginal utility is not linear and one can still obtain a closed form solution is when the utility function exhibits constant absolute risk aversion.

\[ U(c_t) = -1/k \exp\{-kc_t\} \]

- Caballero (1991) shows that with certain finite life, CARA preferences and \( \beta(1 + R) = 1 \), the optimal consumption age profile is flat with no uncertainty, but increasing with income uncertainty.
The Life Cycle Model: Precautionary savings.

- A case when the marginal utility is not linear and one can still obtain a closed form solution is when the utility function exhibits constant absolute risk aversion.

\[ U(c_t) = -\frac{1}{k} \exp\{-kc_t\} \]

- Caballero (1991) shows that with certain finite life, CARA preferences and \( \beta(1 + R) = 1 \), the optimal consumption age profile is flat with no uncertainty, but increasing with income uncertainty.

- This change in the slope of the consumption profile is labeled as precautionary saving and requires prudence (\( U''' > 0 \)).
The Life Cycle Model: Precautionary savings.

- A case when the marginal utility is not linear and one can still obtain a closed form solution is when the utility function exhibits constant absolute risk aversion.

\[ U(c_t) = -\frac{1}{k} \exp\{-kc_t\} \]

- Caballero (1991) shows that with certain finite life, CARA preferences and \( \beta(1 + R) = 1 \), the optimal consumption age profile is flat with no uncertainty, but increasing with income uncertainty.

- This change in the slope of the consumption profile is labeled as precautionary saving and requires prudence (\( U''' > 0 \)).

- The presence and size of precautionary savings is a matter of great relevance for public policy, in so far as public insurance schemes covering such risks as unemployment, health and longevity should reduce the need for consumers to accumulate assets.
The Life Cycle Model: CRRA preferences.

- Quadratic and CARA preferences have undesirable features.
  - Quadratic preferences have a bliss point and certainty equivalence.
The Life Cycle Model: CRRA preferences.

- Quadratic and CARA preferences have undesirable features.
  - Quadratic preferences have a bliss point and certainty equivalence.
  - CARA preferences can imply negative consumption and sound implausible description of risk attitudes.
The Life Cycle Model: CRRA preferences.

- Quadratic and CARA preferences have undesirable features.
  - Quadratic preferences have a bliss point and certainty equivalence.
  - CARA preferences can imply negative consumption and sound implausible description of risk attitudes.
- Attractive alternative: Constant Relative Risk Aversion.

\[ U(c_t) = \frac{1}{1 - \gamma} (c_t^{1-\gamma} - 1), \quad \gamma > 0. \]
Lecture 1
Away from quadratic preferences: precautionary savings.

The Life Cycle Model: CRRA preferences.

- Quadratic and CARA preferences have undesirable features.
  - Quadratic preferences have a bliss point and certainty equivalence.
  - CARA preferences can imply negative consumption and sound implausible description of risk attitudes.

- Attractive alternative: Constant Relative Risk Aversion.

\[ U(c_t) = \frac{1}{1-\gamma}(c_t^{1-\gamma} - 1), \quad \gamma > 0. \]

- This implies

\[ U'(c_t) = c_t^{-\gamma} \]
Under CRRA preferences, the Euler equation is given by:

$$c_t^{-\gamma} = E_t \left[ \beta \left( 1 + R_t^* \right) c_{t+1}^{-\gamma} \right]$$
The Euler equation with CRRA preferences.

- Under CRRA preferences, the Euler equation is given by:

\[ c_t^{-\gamma} = E_t \left[ \beta (1 + R_{t+1}^*) c_{t+1}^{1-\gamma} \right] \]

- This can be re-written as:

\[ E_t \left[ \beta (1 + R_{t+1}^*) \frac{c_{t+1}^{1-\gamma}}{c_t^{-\gamma}} \right] = 1. \]
The Euler equation with CRRA preferences.

- Under CRRA preferences, the Euler equation is given by:

\[ c_t^{-\gamma} = E_t \left[ \beta (1 + R^*_{t+1}) c_{t+1}^{-\gamma} \right] \]

- This can be re-written as:

\[ E_t \left[ \beta (1 + R^*_{t+1}) \frac{c_{t+1}}{c_t^{-\gamma}} \right] = 1. \]

- Or:

\[ \left[ \beta (1 + R^*_{t+1}) \frac{c_{t+1}}{c_t^{-\gamma}} \right] = \epsilon_{t+1}, \quad E_t[\epsilon_{t+1}] = 1. \]
The Euler equation with CRRA preferences.

- Euler equations let researchers be agnostic about the stochastic environment faced by the consumer:
  - the time horizon;
  - the presence of a bequest motive;
  - imperfections in financial markets (as long as there is at least one asset that the consumer can freely trade);
  - frictions in other variables affecting utility.

---

Hall, 1978: first test of the model using Euler equation, quadratic utility, and fixed interest rate with $\beta(1+R) = 1$. Without having to solve the dynamic optimization problem for consumption it allows:

- estimation of preference parameters;
- test the validity of the model.
The Euler equation with CRRA preferences.

- Euler equations let researchers be agnostic about the stochastic environment faced by the consumer:
  - the time horizon;
  - the presence of a bequest motive;
  - imperfections in financial markets (as long as there is at least one asset that the consumer can freely trade);
  - frictions in other variables affecting utility.

- All relevant information is summarized in the level of the marginal utility of wealth.
The Euler equation with CRRA preferences.

- Euler equations let researchers be agnostic about the stochastic environment faced by the consumer:
  - the time horizon;
  - the presence of a bequest motive;
  - imperfections in financial markets (as long as there is at least one asset that the consumer can freely trade);
  - frictions in other variables affecting utility.

- All relevant information is summarized in the level of the marginal utility of wealth.

- The Euler equation is very useful from an empirical point of view.
  - Hall, 1978: first test of the model using Euler equation, quadratic utility, and fixed interest rate with $\beta(1 + R) = 1$.

- Without having to solve the dynamic optimization problem for consumption it allows:
  - estimation of preference parameters;
  - test the validity of the model
The Life Cycle Model: Discussion.

- We have considered the problem in terms of total financial wealth (and cash in hand), which has a return of $R^*_t$. 

These relationships are useful to study asset pricing. Notice that we have assumed no limits on the assets, except that at the end of life they cannot be negative.
The Life Cycle Model: Discussion.

- We have considered the problem in terms of total financial wealth (and cash in hand), which has a return of $R_t^*$.
- However, we could have considered the problem of determining the optimal portfolio and consider the return for each different asset.
The Life Cycle Model: Discussion.

- We have considered the problem in terms of total financial wealth (and cash in hand), which has a return of $R_t^*$. 
- However, we could have considered the problem of determining the optimal portfolio and consider the return for each different asset. 
- We would then derive an Euler equation for each asset $i$:

$$
\frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta(1 + R_{t+1}^i) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right], \quad \forall i.
$$

These relationships are useful to study asset pricing. Notice that we have assumed no limits on the assets, except that at the end of life they cannot be negative. This implies that people can borrow in each of the assets considered.
The Life Cycle Model: Discussion.

- We have considered the problem in terms of total financial wealth (and cash in hand), which has a return of \( R_t^* \).
- However, we could have considered the problem of determining the optimal portfolio and consider the return for each different asset.
- We would then derive an Euler equation for each asset \( i \):

\[
\frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta (1 + R_{t+1}^i) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right], \quad \forall i.
\]

- These relationships are useful to study asset pricing.
The Life Cycle Model: Discussion.

- We have considered the problem in terms of total financial wealth (and cash in hand), which has a return of $R_t^*$.  
- However, we could have considered the problem of determining the optimal portfolio and consider the return for each different asset.  
- We would then derive an Euler equation for each asset $i$:  
  
  $$
  \frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta (1 + R_{t+1}^i) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right], \quad \forall i.
  $$

- These relationships are useful to study asset pricing.  
- Notice that we have assumed no limits on the assets, except that at the end of life they cannot be negative.
The Life Cycle Model: Discussion.

- We have considered the problem in terms of total financial wealth (and cash in hand), which has a return of $R_t^\ast$.
- However, we could have considered the problem of determining the optimal portfolio and consider the return for each different asset.
- We would then derive an Euler equation for each asset $i$:
  \[
  \frac{\partial U(c_t)}{\partial c_t} = E_t \left[ \beta (1 + R_{t+1}^i) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right], \quad \forall i.
  \]
- These relationships are useful to study asset pricing.
- Notice that we have assumed no limits on the assets, except that at the end of life they cannot be negative.
- This implies that people can borrow in each of the assets considered.
The Life Cycle Model: Discussion.

- If there is one (or more) asset for which such limits exist and they are binding, the Euler equation will not hold for that asset.
The Life Cycle Model: Discussion.

- If there is one (or more) asset for which such limits exist and they are binding, the Euler equation will not hold for that asset.
- However, it will hold for any asset for which the consumer is not at a corner, even if there are are other assets for which is constrained.
- For these assets we will have:

\[
\frac{\partial U(c_t)}{\partial c_t} > E_t \left[ \beta (1 + R_{t+1}^k) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right].
\]
We can also consider restrictions on total wealth:

$$W_t \geq 0, \quad \forall t$$
The Life Cycle Model: Liquidity constraints.

- We can also consider restrictions on total wealth:
  \[ W_t \geq 0, \; \forall t \]

- This means that the consumer cannot borrow.
The Life Cycle Model: Liquidity constraints.

- We can also consider restrictions on total wealth:
  \[ W_t \geq 0, \quad \forall t \]

- This means that the consumer cannot borrow.
- Or we can assume that she can only borrow limited amounts:
  \[ W_t \geq -B, \quad \forall t \]

- We will then have:
  \[
  \frac{\partial U(c_t)}{\partial c_t} \geq E_t \left[ \beta (1 + R_t^{*}) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right].
  \]
  where the inequality will be strict when the restriction is binding.
The Life Cycle Model: Liquidity constraints.

- We can also consider restrictions on total wealth:
  \[ W_t \geq 0, \forall t \]

- This means that the consumer cannot borrow.
- Or we can assume that she can only borrow limited amounts:
  \[ W_t \geq -\overline{B}, \forall t \]

- We will then have:
  \[ \frac{\partial U(c_t)}{\partial c_t} \geq E_t \left[ \beta (1 + R_{t+1}^*) \frac{\partial U(c_{t+1})}{\partial c_{t+1}} \right]. \]
  where the inequality will be strict when the restriction is binding.
- This is the case of liquidity constraints.
- Individuals will like to increase current consumption by moving resources from the future but they cannot.
The Life Cycle Model: Natural liquidity constraint.

- There is an end-point constraint on net worth that must hold with unit probability, that is the consumer has to die without debt, or pay back all her debt with probability one.
- This can impose important limitations to the ability to smooth consumption.
The Life Cycle Model: Natural liquidity constraint.

- There is an end-point constraint on net worth that must hold with unit probability, that is the consumer has to die without debt, or pay back all her debt with probability one.
- This can impose important limitations to the ability to smooth consumption.
- If the income process is not bounded away from zero and the marginal utility of consumption tends to infinity at very low levels of consumption, then the consumer will never want to borrow in such a situation.
The Life Cycle Model: Natural liquidity constraint.

- There is an end-point constraint on net worth that must hold with unit probability, that is the consumer has to die without debt, or pay back all her debt with probability one.
- This can impose important limitations to the ability to smooth consumption.
- If the income process is not bounded away from zero and the marginal utility of consumption tends to infinity at very low levels of consumption, then the consumer will never want to borrow in such a situation.
- Even if the income process is bounded away from zero, the consumer will not want to borrow more than the present value of the lowest level of income.
The Life Cycle Model: Natural liquidity constraint.

- There is an end-point constraint on net worth that must hold with unit probability, that is the consumer has to die without debt, or pay back all her debt with probability one.
- This can impose important limitations to the ability to smooth consumption.
- If the income process is not bounded away from zero and the marginal utility of consumption tends to infinity at very low levels of consumption, then the consumer will never want to borrow in such a situation.
- Even if the income process is bounded away from zero, the consumer will not want to borrow more than the present value of the lowest level of income.
- Similar considerations apply whenever the survival probability is less than one, if longevity risks cannot be fully insured.
We have sketched the essence of the life cycle model for consumption.
Using the model

- We have sketched the essence of the life cycle model for consumption.
- Even before considering a number of extensions that can make it more realistic/ useful we can put it to work.
Using the model

- We have sketched the essence of the life cycle model for consumption.
- Even before considering a number of extensions that can make it more realistic/useful we can put it to work.
- We will review how this simple structure can frame some of the puzzle we discussed when motivating the model.
Using the model

- We have sketched the essence of the life cycle model for consumption.
- Even before considering a number of extensions that can make it more realistic/ useful we can put it to work.
- We will review how this simple structure can frame some of the puzzle we discussed when motivating the model.
- And/or how it needs to be changed to explain some of these facts.
Using the model

- We have sketched the essence of the life cycle model for consumption.
- Even before considering a number of extensions that can make it more realistic/useful we can put it to work.
- We will review how this simple structure can frame some of the puzzle we discussed when motivating the model.
- And/or how it needs to be changed to explain some of these facts.
- We will consider:
  - Policy implications: taxation of saving, provision of social security.
  - Labour supply elasticities in macro model
Policy implications

- The role of the life cycle model in a variety of macro-models:
Policy implications

- The role of the life cycle model in a variety of macro-models:
  - Overlapping generations
  - Generational accounting
  - Relationship between demographic changes and aggregate saving rates.

The effect of the provision of public pensions (funded and unfunded).

What is the effect of the taxation of saving?

The answer depends crucially on the size of the intertemporal elasticity of substitution.

While the answer is theoretically straightforward, to quantify its size requires some hard evidence on structural parameters.

The effectiveness of incentives to raise savings depends on the nature of the answer.
Policy implications

- The role of the life cycle model in a variety of macro-models:
  - Overlapping generations
  - Generational accounting
  - Relationship between demographic changes and aggregate saving rates.
- The effect of the provision of public pensions (funded and unfunded).

The answer depends crucially on the size of the intertemporal elasticity of substitution. While the answer is theoretically straightforward, to quantify its size requires some hard evidence on structural parameters. The effectiveness of incentives to raise savings depends on the nature of the answer.
Policy implications

- The role of the life cycle model in a variety of macro-models:
  - Overlapping generations
  - Generational accounting
  - Relationship between demographic changes and aggregate saving rates.
- The effect of the provision of public pensions (funded and unfunded).
- What is the effect of the taxation of saving?
Policy implications

- The role of the life cycle model in a variety of macro-models:
  - Overlapping generations
  - Generational accounting
  - Relationship between demographic changes and aggregate saving rates.
- The effect of the provision of public pensions (funded and unfunded).
- What is the effect of the taxation of saving?
  - The answer depends crucially on the size of the intertemporal elasticity of substitution.
Policy implications

- The role of the life cycle model in a variety of macro-models:
  - Overlapping generations
  - Generational accounting
  - Relationship between demographic changes and aggregate saving rates.

- The effect of the provision of public pensions (funded and unfunded).

- What is the effect of the taxation of saving?
  - The answer depends crucially on the size of the intertemporal elasticity of substitution.
  - While the answer is theoretically straightforward, to quantify its size requires some hard evidence on structural parameters.
Policy implications

- The role of the life cycle model in a variety of macro-models:
  - Overlapping generations
  - Generational accounting
  - Relationship between demographic changes and aggregate saving rates.

- The effect of the provision of public pensions (funded and unfunded).

- What is the effect of the taxation of saving?
  - The answer depends crucially on the size of the intertemporal elasticity of substitution.
  - While the answer is theoretically straightforward, to quantify its size requires some hard evidence on structural parameters.
  - The effectiveness of incentives to raise savings depends on the nature of the answer.
Changes in assets to changes in interest rate from 2% to 2.5%.

- Attanasio and Wakefield (2008): how do asset change in reaction to changes in the interest rate as the E.I.S. changes?
- A model of a single consumer with an isoelastic utility function (they take $EIS=1$ as baseline), no bequest motive and a stochastic income process calibrated on UK data.
Life cycle profiles: changes in assets to changes in interest rate from 2% to 2.5%.

Attanasio and Wakefield also explore the role of demographics, and find they reduce responses to interest rate changes.