The empirics of the life cycle model.

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Outline of Lecture 3: The empirics of the life cycle model.

1. Measuring consumption: some serious challenges.

2. Intra-temporal conditions:
   1. Demand Analysis.
   2. Marginal Rates of Substitutions for Labour supply.

3. Approaches based on Euler equations.
   - Aggregation and estimation of preferences.

4. Approaches based on the level of consumption:
   - Calibration and simulation methods.

5. The evolution of second moments of consumption and earnings.
The challenge of measuring consumption

- Microeconomic data that include information on consumption are essential for a variety of purposes:
  - Computing the weights for CPI.
  - Estimating the models we have been discussing.
  - Evaluating a variety of policies/
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- However:
  - In many countries the information is collected only when weights are revised (roughly every 10 years).
  - Very few countries have a longitudinal dimension (at the household level).
    - US: 4 quarters
    - Spain: 8 quarters.
  - Often data are affected by severe measurement error.
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- It will be valuable to cross the information on consumption with information about other variables:
  - Savings (including pension rights);
  - Health;
  - Stocks of durables.
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- The model represents equilibrium conditions. This means that the choice variables (the quantities) can be related to taste shocks.
- In some cases, even individual prices could be related to individual taste shocks.
  - This is the case for wages.
  - Taste for work and ability (and productivity) could be related.
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Caveat: skill levels could be related to tastes. We might need to control for skill level in the MRS.
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How do we deal with measurement error?

How do we enter observed heterogeneity?
  - Intercept shifts.
  - Interaction with prices.
  - Slope shifts?
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- The Euler equation implies that no variable known to the consumer at time \( t \) should help predict the change in consumption between \( t \) and \( t+1 \).
Estimating Euler equations.

\[ \beta(1 + R_{t+1}) \frac{MU_{c_{t+1}}}{MU_{c_t}} - 1 = \epsilon_{t+1} \]

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- Estimation of the Euler equation requires observations covering a long period of time (Chamberlain, 1984, Hayashi, 1987).
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- There is no reason why expectational errors should averaged out to zero over the cross section.
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  - unless all individuals have the same 'surprise'.
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\begin{align*}
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$$ E_N[(\epsilon - d_t)z^i_t] \neq 0 $$
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- In the case of micro data, it is not necessary to have a large T for each household (or group of households).
- It is possible to work with unbalanced panels, as long as they cover a sufficiently long time horizon.
- Additional considerations need to be kept in mind about the nature of the other residuals.
  - Measurement error.
  - Taste shifters.
  - Unobserved heterogeneity.
The Euler equation with CRRA preferences.

Under CRRA preferences, the Euler equation is given by:

\[ c_t^{-\gamma} = E_t \left[ \beta \left(1 + R^*_{t+1}\right) c_{t+1}^{-\gamma} \right] \]
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or:

\[ \left[ \beta (1 + R_{t+1}^*) \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \right] = \epsilon_{t+1}, \quad E_t[\epsilon_{t+1}] = 1. \]
Log-linearizing the CRRA Euler equation.

- The Euler equation can be conveniently log-linearized.

\[ \Delta \log(c_{t+1}) = \alpha_{t+1} + \frac{1}{\gamma} \log(1 + R_{t+1}^k) + \tilde{\epsilon}_{t+1} \]

where \( \epsilon_{t+1} = \log(\tilde{\epsilon}_{t+1}) \)
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- This is counteracted by an ”income effect” since with a higher interest rate: a given target level of future consumption is achieved with less saving.
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- With more than two periods there is also a wealth effects that reinforces the substitution effect as expected future incomes are discounted with higher interest rates.
- What is the interpretation of \( \alpha_{t+1} \)?
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- With more than two periods there is also a wealth effect that reinforces the substitution effect as expected future incomes are discounted with higher interest rates.
- What is the interpretation of \( \alpha_{t+1} \)?
- To answer this question we want to go back to the basic Euler equation.
Log-linearizing the CRRA Euler equation.

Remember that we started from:

\[
\beta(1 + R_{t+1}^*) \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} = \tilde{\epsilon}_{t+1}, \quad E_t[\tilde{\epsilon}_{t+1}] = 1.
\]
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Remember that we started from:

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\beta (1 + R_{t+1}^*) \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} = \tilde{e}_{t+1}, \quad E_t[\tilde{e}_{t+1}] = 1.
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- The 'residual' term \( \tilde{e}_{t+1} \) represent an expectational error. Under rational expectations it cannot be predicted with the information available at time \( t \).
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\]

- The 'residual' term $\tilde{\epsilon}_{t+1}$ represent an expectational error. Under rational expectations it cannot be predicted with the information available at time $t$.
- It is a non-negative term as all the components of the left-hand side are non negative.
- If it is bounded away from zero we can take logs.
Log-linearizing the CRRA Euler equation.

Remember that we started from:

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\beta(1 + R^*_t) \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} = \tilde{\epsilon}_{t+1}, \quad E_t[\tilde{\epsilon}_{t+1}] = 1.
\]

- The 'residual' term \( \tilde{\epsilon}_{t+1} \) represent an expectational error. Under rational expectations it cannot be predicted with the information available at time \( t \).
- It is a non-negative term as all the components of the left-hand side are non-negative.
- If it is bounded away from zero we can take logs.
- By Jensen inequality, while \( \log(E_t[\tilde{\epsilon}_{t+1}]) = 0 \),
  \[ E_t[\log(\tilde{\epsilon}_{t+1})] \neq 0. \]
Log-linearizing the CRRA Euler equation.

Let us assume that \((1 + R^*_{t+1})\) and \(\frac{c_{t+1}}{c_t}\) are jointly log normal:

\[
\begin{pmatrix}
\log \left( \frac{c_{t+1}}{c_t} \right) \\
\log(1 + R^*_{t+1})
\end{pmatrix}
\sim N\left( \begin{pmatrix}
\mu_c \\
\mu_R
\end{pmatrix}, \begin{pmatrix}
\sigma^2_{c_t} & \sigma_{c_tR_t} \\
\sigma_{c_tR_t} & \sigma^2_{R_t}
\end{pmatrix} \right)
\]
Log-linearizing the CRRA Euler equation.

Let us assume that \( (1 + R_{t+1}^*) \) and \( \frac{c_{t+1}}{c_t} \) are jointly log normal:

\[
\begin{pmatrix}
\log \left( \frac{c_{t+1}}{c_t} \right) \\
\log \left( 1 + R^*_{t+1} \right)
\end{pmatrix} 
\sim \mathcal{N}
\begin{pmatrix}
\mu_c \\
\mu_R
\end{pmatrix},
\begin{pmatrix}
\sigma^2_c & \sigma_{c t} R_t \\
\sigma_{c t} R_t & \sigma^2_{R t}
\end{pmatrix}
\]

This implies that \( \tilde{\epsilon}_{t+1} \) is also log-normal:

\[
\log (\tilde{\epsilon}_{t+1}) \sim \mathcal{N}(\mu_e, \sigma^2_{e t})
\]

where:

\[
\mu_{e t} = \log(\beta) + \mu_{R t} - \gamma \mu_{c t}
\]
\[
\sigma^2_{e t} = \gamma^2 \sigma^2_c + \sigma^2_{R t} - 2 \gamma \sigma_{c t} R_t
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Log-linearizing the CRRA Euler equation.

It therefore follows that:

$$\Delta \log(c_{t+1}^h) = \alpha_{t+1} + \frac{1}{\gamma} \log(1 + R_{t+1}^k) + \epsilon_{t+1}^h$$

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- Specification.
Why estimate an Euler equation?

- Estimate preference parameters.
Why estimate an Euler equation?

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- Test the model.
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Why not.
- It is not a consumption function.
- Cannot be used to predict consumption as a function of changes to policy.
The residuals of the (log-linearized) equation are expectation errors. They reflect expectations errors in:
- Consumption growth;
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The nature of the residuals and estimation techniques

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  - Consumption growth;
  - Interest rates.
- This implies that they are correlated with actual realizations (of consumption and interest rates).
- This makes OLS estimates inconsistent.
- IV or GMM techniques are however readily available.
  - Any available to consumers at time $t$ us a valid instrument.

$$E[z_t^h \epsilon_{t+1}^h] = 0$$
- The abundance of instruments allows over-identification.
The nature of the residuals and estimation techniques

- IV or GMM techniques are based on the assumption that a moment population is equal to zero.

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- Even if we use micro panel data, to achieve consistency, we need large T.
The $\alpha_{t+1}$: is it a problem?

\[ \Delta \log(c^h_{t+1}) = \alpha_{t+1} + \frac{1}{\gamma} \log(1 + R^k_{t+1}) + \epsilon^h,k_{t+1} \]

\[ \alpha_{t+1} = \log(\beta) + \frac{1}{2} [\gamma^2 \sigma^2_c + \sigma^2_R - 2\gamma \sigma_c R_t] \]

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  - Simulations seem to suggest it is ok. (Attanasio and Low (2003)).
Non-linear Euler equations.

Why not estimate the non-linear version of the Euler equation?

\[ E_t \left[ \beta \left( 1 + R_{t+1}^* \right) \frac{C_{t+1}^{\gamma - 1}}{C_t^{\gamma - 1}} \right] - 1 = 0. \]
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Why not estimate the non-linear version of the Euler equation?

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E_t \left[ \beta(1 + R_{t+1}^*) \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \right] - 1 = 0.
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\[
g_t(x_{t+1}, z_t, \theta) \equiv \left( \left[ \beta(1 + R_{t+1}^*) \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \right] - 1 \right) \cdot z_t
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where \( z_t \) is a \( k \times 1 \) vector of variables known at time \( t \), \( x_{t+1} \) represents a vector of data and \( \theta \) a vector of parameters.
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$$\hat{\theta} = \text{argmin} [G'_T W G_T]$$

where $G_T = \frac{1}{T} \sum_{t=1}^{T} g_t$ and $W$ is a positive definite matrix.
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Aggregation

The Euler equation was derived from the maximization problem of a single consumer.

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- This implies it cannot be estimated using aggregate data.

\[ C_t = \frac{1}{H} \sum_h c_t^h \]

\[ \log(C_t) = -\log(H) + \log(\sum_h c_t^h) \]
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Aggregating the Euler equation one gets:

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While on aggregate data:

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- \( \nu_t \) is a measure of inequality and is sometimes referred to as the Theil entropy measure.
- If such a variable is correlated with the instrument used in estimation it will induce biases in the estimation and possible rejections of the model.
Estimating on properly aggregated data.

\[ \Delta \log(c_{t+1}^h) = \alpha + \frac{1}{\gamma} \log(1 + R_{t+1}^k) + \epsilon_{t+1}^{h,k} \]

- This equation should be estimated on panel data.
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  \[ \overline{lc}_{g} = \frac{1}{H_{g}^t} \sum_{h \in g} \log(c_{t}^h). \]
- Aggregating the euler equation over the households belonging to group \( g \) we have:
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- One controls the aggregation process directly.
- Econometric caveats:
  - Small sample induces MA errors.
  - Varying membership.
  - Non linearities and interactions.
We mentioned several times that to bring the model to the data we need to make it realistic.
Specification

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  - Observable variables.
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Example:

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U(c_t, z_t, v_t) = \frac{(c_t^h)^{1-\gamma}}{1-\gamma} \exp\{\theta' z_t + u_t\}
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$$U(c_t, z_t, v_t) = \frac{(c^h_t)^{1-\gamma}}{1-\gamma} \exp\{\theta' z_t + u_t\}$$

The implied (log-linearized) Euler equation is:

$$\Delta \log (c^h_{t+1}) = \alpha_{t+1} + \frac{1}{\gamma} \left[ \log (1 + R^k_{t+1}) + \theta' \Delta z_{t+1} + \Delta u_{t+1} \right] + \epsilon_{t+1}^{h,k}$$
Specification

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Examples of observable variables:

- Demographics.
- Labour supply variables (including discrete variables - participation).
- Seasonal effects.
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Unobservable variables:

- The properties of the estimators depend on the properties of the unobservable process:
  - Fixed effects \(\rightarrow\) drop out of the equation.
  - Random walk \(\rightarrow\) additional white noise.
  - White noise \(\rightarrow\) MA(1) residuals.
Euler equations: empirical evidence

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- But a low response of consumption growth to the real interest rate could obtain if some consumers are liquidity constrained, or if the error term correlates with the part of the interest rate explained by the instruments.
- Attanasio and Weber (1993, 1995) stress aggregation bias can be responsible for such correlation. They use cohort data.
- When they focus on cohorts of individuals who are least likely to be liquidity constrained, and control for changes in taste shifters, they estimate an elasticity around 0.8 using both UK and US data.

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The price we pay is the complete specification of the stochastic environment where individuals are assumed to live.
Solving the life cycle model.

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- Non stationary problems, for instance because of the presence of non stationary income, can be re-written so to be stationary.
- In the finite horizon case you cannot have stationary rules.
- Typical solution is by backward induction.
Solving the life cycle model: an example with infinite horizon.

Consider the following problem:

\[
\begin{align*}
\text{Max } & \quad E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}) \\
A_{t+1} & = (1 + R_{t+1})(A_t + Y_t - C_t) \\
U(C) & = \frac{C^{1-\gamma}}{1-\gamma}, \quad \text{if } \gamma > 0, \gamma \neq 1; \\
U(C) & = \ln(C), \quad \text{if } \gamma = 1 \\
Y_t & = P_t U_t, \quad U_t \text{ i.i.d.} \\
P_t & = T_t P_{t-1} V_t, \quad T_t \text{ deterministic}, \quad V_t \text{ i.i.d.}
\end{align*}
\]
Solving the life cycle model: an example with infinite horizon.

In terms of cash in hand \( X_t = A_t + Y_t \):

\[
\text{Max} \quad E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j})
\]

\( X_{t+1} = (1 + R_{t+1})(X_t - C_t) + Y_{t+1}, \)

\( U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad \text{if } \gamma > 0, \gamma \neq 1; \)

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Solving the life cycle model: an example with infinite horizon.

Normalize by permanent income:

\[
\text{Max } E_t \sum_{j=0}^{\infty} \beta^j U(c_{t+j} \cdot P_t) \quad c_t = C_t / P_t
\]

\[
x_{t+1} = (1 + R_{t+1})(x_t - c_t) \frac{P_t}{P_{t+1}} + y_{t+1}, \quad x_t = X_t / P_t, \quad y_t = Y_t / P_t
\]

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  \[ c_t = h(x_t) \]
- Notice that the rule does not depend on $t$.
- This result is possible because it is possible to parametrize the problem so that:
  \[
  r_t(x_t, u_t) = \beta^t r(x_t, u_t) \\
  g_t(x_t, u_t) = g(x_t, u_t)
  \]
Solving the life cycle model: an example with infinite horizon.

- Further simplification follows from the fact that our problem can be parametrized so that:
  \[ \frac{\partial g}{\partial x} = 0 \]

- Note in our problem we can write \( x_t \) as the state variable and \( u_t \equiv x_t - c_t \) as the control, so that:
  \[ x_{t+1} = (1 + R_{t+1})u_t + y_{t+1} \]
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  \[
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  \]

This delivers the result that:
\[
V' = \frac{\partial r}{\partial x}
\]
Solving the life cycle model: an example with finite horizon.

Consider again a similar problem but with finite lives.

\[
\text{Max } E_t \sum_{j=0}^{T} \beta^j U(c_{t+j} \cdot P_t) \quad c_t = C_t / P_t
\]

\[
x_{t+1} = (1 + R_{t+1})(x_t - c_t) \frac{P_t}{P_{t+1}} + y_{t+1}, \quad x_t = X_t / P_t, \quad y_t = Y_t / P_t
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  1. Start from time $T$: $c_T = x_T$;
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\[
c_{T-1}^{-\gamma} = E_t \left[ \beta (1 + R_T) c_T^{-\gamma} \right]
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\]

3. Substitute for the intertemporal budget constraint and solve for $c_{T-1}$ as a function of $x_{T-1}$:

\[
  c_T^{\gamma} = E_t \left[ \beta (1 + R_T) (x_{T-1} - c_{T-1} + y_T) (1 + R_T) \right]^{\gamma}
\]

\[
  c_{T-1} = h_{T-1} (x_{T-1})
\]
Solving the life cycle model:
an example with finite horizon.

\[ c_{T-1} = h_{T-1}(x_{T-1}) \]

- This equation can be solved for each of many values of \( x \).
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- Once we have the solution for \( c_{T-1} \) for several values of \( x_{T-1} \) we can consider the Euler equation between \( T - 2 \) and \( T - 2 \).

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Solving the life cycle model: an example with finite horizon.

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\[ c_{T-2}^{\gamma} = E_t \left[ \beta(1 + R_{T-1})c_{T-1}^{\gamma} \right] = E_t \left[ \beta(1 + R_{T-1})[h_{T-1}(x_{T-1})]^{\gamma} \right] \]
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\]

\[
= E_t[\beta(1 + R_{T-1})[h_{T-1}((x_{T-2} - c_{T-2} + y_{T-1})(1 + R_{T-1}))]^{-\gamma}]
\]
Solving the life cycle model: an example with finite horizon.

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One difficulty arises from the fact that the points for \( x_{T-1} \) at which we need to evaluate the function \( h_{T-1} \) are not necessarily those for which we solved the equation in the previous step.
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- One difficulty arises from the fact that the points for \( x_{T-1} \) at which we need to evaluate the function \( h_{T-1} \) are not necessarily those for which we solved the equation in the previous step.
- It will be necessary to interpolate.
- The position of the grid is crucial to obtain numerically precise solutions.
Solving the life cycle model: 
Humps and Bumps.

- Attanasio, Banks, Meghir and Weber (1999) solve a version of the life cycle model where some parameters are estimated from data.
  - Elasticity of intertemporal substitution.
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  - but...no labour supply.

- Consider different education groups and calibrate different discount rates.
Evolution of second moments

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Conversely, the evolution of variances (and covariances) is informative of the fraction of shocks that is reflected into consumption and what instead is insured.
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- The former reflects only permanent shocks, the latter permanent and transitory shocks.
- Conversely, the evolution of variances (and covariances) is informative of the fraction of shocks that is reflected into consumption and what instead is insured.
- Under perfect insurance, the cross sectional variance of consumption is constant over time.
Evolution of second moments

Consider the following set up.

\[ \log(Y_{i,t}) = \phi' X_{i,t} + P_{i,t} + v_{i,t} \]
\[ P_{i,t} = P_{i,t} + \zeta_{i,t} \]
\[ v_{i,t} = \sum_{j=0}^{q} \theta_j \epsilon_{i,t-j} \]
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\[
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\[
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\[
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\[ \Delta y_{i,t} = \zeta_{i,t} + \Delta v_{i,t} \]

\[ \Delta c_{i,t} = \phi_t \zeta_{i,t} + \psi_t \epsilon_{i,t} + \zeta_{i,t} \]
Evolution of second moments

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We can use second moments and their evolution to estimate the 'insurance' parameters and test these hypotheses.
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Evolution of second moments

\[ \text{Var}(\Delta y_{i,t}) = \text{var}(\zeta_{i,t}) + \text{var}(\Delta v_{i,t}) \]

\[ \text{Cov}(\Delta y_{i,t}, \Delta y_{i,t-s}) = \text{Cov}(\Delta v_{i,t}, \Delta v_{i,t-s}) \]
Evolution of second moments

\begin{align*}
Var(\Delta y_{i,t}) &= var(\zeta_{i,t}) + var(\Delta v_{i,t}) \\
Cov(\Delta y_{i,t}, \Delta y_{i,t-s}) &= Cov(\Delta v_{i,t}, \Delta v_{i,t-s}) \\
\Delta Var(c_{i,t}) &= \Delta \phi_t^2 var(\zeta_{i,t}) + \phi_{t-1}^2 var(\zeta_{i,t}) + \Delta \psi_t^2 var(\epsilon_{i,t}) \\
&\quad + \psi_{t-1}^2 \Delta var(\epsilon_{i,t}) \\
Cov(\Delta c_{i,t}, \Delta c_{i,t-s}) &= \phi_t^2 var(\zeta_{i,t}) + \psi_t^2 Var(\epsilon_{i,t}) + var(\zeta_{i,t})
\end{align*}
Evolution of second moments

Table 7—Minimum-Distance Partial Insurance and Variance Estimates

<table>
<thead>
<tr>
<th>Consumption:</th>
<th>Nondurable net income baseline</th>
<th>Nondurable earnings only baseline</th>
<th>Nondurable male earnings baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.6423</td>
<td>0.3100</td>
<td>0.2245</td>
</tr>
<tr>
<td>(Partial insurance perm. shock)</td>
<td>(0.0945)</td>
<td>(0.0574)</td>
<td>(0.0493)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.0533</td>
<td>0.0633</td>
<td>0.0502</td>
</tr>
<tr>
<td>(Partial insurance trans. shock)</td>
<td>(0.0435)</td>
<td>(0.0309)</td>
<td>(0.0294)</td>
</tr>
</tbody>
</table>

Notes: This table reports DWMD results of the parameters of interest. We also estimate time-varying variances of measurement error in consumption (results not reported for brevity). See the main text for details. Standard errors in parentheses.